Numerical Optimization

Homework 5

Due 14.07.2014

Give your answers with logical and/or mathematical explanations. Hand-in your homework in the beginning of a lecture on due date. Late submissions will not be accepted. Assigned points are shown in square brackets, which will be re-scaled so that the total homeworks points will be 40.

1.[5] Let $h : \mathbb{R}^n \to \mathbb{R}^m$ be a smooth vector-valued function (each component function $h_i : \mathbb{R}^n \to \mathbb{R}$ is continuously differentiable for $i = 1, 2, \ldots, m$.). Consider the (in general nonsmooth) unconstrained optimization problems

$$\min_{x \in \mathbb{R}^n} \|h(x)\|_\infty, \text{ and } \min_{x \in \mathbb{R}^n} \max_{i=1,\ldots,m} h_i(x)$$

Reformulate each of these problems as a smooth constrained optimization problem.

2.[5] Formulate and solve the following Euclidean projection problem: find the point x with the smallest Euclidean norm in a half-space $H := \{x \in \mathbb{R}^n : a^T x + b \geq 0\}$ where $a \in \mathbb{R}^n$, $a \neq 0$, and $b \in \mathbb{R}$ are given. Hint: minimize $\| \cdot \|_2^2$ instead of $\| \cdot \|_2$ to have the same solution.

3.[10] Consider the problem of finding the point $(x, y)^T \in \mathbb{R}^2$ on the parabola $y = (x-1)^2/5$ that is closest to $(1, 2)^T$, in the Euclidean norm:

$$\min_{(x,y)^T \in \mathbb{R}^2} (x-1)^2 + (y-2)^2, \text{ subjet to } (x-1)^2 = 5y.$$

(a) Find all the point satisfying the KKT conditions. Is the LICQ satisfied?

(b) Which of these are solutions?

(c) By directly substituting the constraint into the objective function and eliminating the variable x, an unconstrained minimization problem is obtained. Show that the solutions of this problem cannot be solutions of the original problem. What was wrong with the direct substitution?

4.[5] Verify that the dual of the following linear program

$$\min_{x \in \mathbb{R}^n} c^T x \text{ s.t. } Ax \geq b, \ x \geq 0,$$

for given $A \in \mathbb{R}^{m \times n}$, $c \in \mathbb{R}^n$, and $b \in \mathbb{R}^m$ can expressed as

$$\max_{\lambda \in \mathbb{R}^m} b^T \lambda \text{ s.t. } A^T \lambda \leq c, \ \lambda \geq 0.$$